Quantum Mechanics

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Separation of Variables



Separable Equations of Order-One

The simplest ODE of order-one have the following form

$$P(x)dx + Q(y)dy = 0$$

The solution is found by simply integrating.

Example 1

$$\frac{dy}{dx} = -y^2 e^x$$

can be rewritten as

$$-\frac{dy}{y^2} = e^x dx$$

Each side can be integrated separately. Thus

$$\frac{1}{y} = e^x + c$$





General and Particular Solutions

Example 2

$$2x(y+1)dx - ydy = 0$$

where y = -2 when x = 0.

Solution:

Put x and y in separate terms,

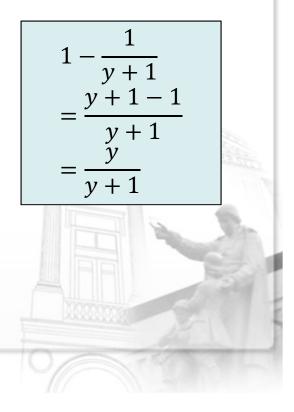
$$2xdx - \frac{ydy}{y+1} = 0$$

This can be recast for easier integration

$$2xdx = \left(1 - \frac{1}{y+1}\right)dy$$

Integrating each side yields the general solution

$$x^2 = y - \ln|y+1| + c$$





General and Particular Solutions

Example 2 (cont'd)

Applying the initial conditions y = -2 when x = 0,

$$0 = -2 - \ln|-2 + 1| + c$$

indicating that

$$c = 2 + ln1 = 2$$

The particular solution is therefore

$$x^2 = y - ln|y + 1| + 2$$





Change of Variables

Sometimes when the variables are not separable, a differential equation may be reduced to one by an appropriate change of variable. The general form of differential equation that is amenable to this approach is

$$\frac{dy}{dx} = f(ax + by)$$

If we let

$$w = ax + by$$

then

$$\frac{dw}{dx} = a + b\frac{dy}{dx}$$

and the differential equation can be rewritten as

$$\frac{dw}{dx} - a = bf(w)$$

which is separable as

$$\frac{dw}{a+bf} = dx$$





Change of Variables

Example 3

$$\frac{dy}{dx} = 8x + 4y + (2x + y - 1)^2$$

Solution:

Let

Then

$$\frac{dw}{dx} = 2 + \frac{dy}{dx}$$

w = 2x + y

The differential equation can then be recast as

$$\frac{dw}{dx} - 2 = 4w + (w - 1)^2$$

Separating the variables

$$\frac{dw}{[4w + (w - 1)^2 + 2]} = dx$$





Change of Variables

