Quantum Mechanics 2

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Hydrogen Atom

Zeeman Effect

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Normal Zeeman Effect

If a Hydrogen atom is placed in a magnetic field, it interacts with this field through its orbital and spin angular momenta.

$$H_Z = -\mu \cdot B$$

The magnetic moment of an electron due to its orbital angular momentum is

$$\mu_l = -\frac{e}{2m_e} L$$

Thus,

$$H_Z = \frac{e}{2m_e} L \cdot B$$

For a uniform magnetic field, we may take its direction as the $z-$ axis. The first order correction due to this interaction is then

$$E_{(NZ)}^{(1)} = \langle nlm|H_Z|nlm\rangle = \frac{eB}{2m_e}\langle nlm|L_Z|nlm\rangle = \frac{eB}{2m_e} m\hbar$$

The leads to the splitting of the azimuthal degeneracy, and it is called the **Normal Zeeman Effect**

Anomalous Zeeman Effect

The magnetic moment of an electron due to its spin is

$$\mu_s = -\frac{e}{m_e} S$$

Its interaction Hamiltonian is then

$$H_Z = \frac{e}{m_e} S \cdot B$$

Taken together with the interaction with the orbital angular momentum

$$H_Z = \frac{e}{2m_e} (L + 2S) \cdot B$$

For a uniform magnetic field, we may take its direction as the $z -$ axis. The first order correction due to this interaction is then

$$E^{(1)}_{(AZ)} = \langle nlm_l m_s | H_Z | nlm_l m_s \rangle = \frac{eB}{2m_e} \langle nlm_l m_s | (L_z + 2S_z) | nlm_l m_s \rangle$$

$$= \frac{eB}{2m_e} (m_l + 2m_s)\hbar$$

This leads to a doubling of the azimuthal degeneracy splits, and is known as the **Anomalous Zeeman Effect**

Weak Fields

If the external magnetic field is very weak, the Zeeman interaction is much weaker than the spin-orbit interaction. Thus, we may take it as a perturbation relative to the Hamiltonian

$$H_0 = \frac{p^2}{2m_e} - \frac{e^2}{4\pi\varepsilon_0 r} + \frac{e^2}{4\pi\varepsilon_0}\frac{1}{2m_e^2c^2r^3} S \cdot L$$

The good quantum numbers for fine structure are n, l, s, j, m. Thus,

$$E_{(WZ)}^{(1)} = \langle nlsjm|H_Z|nlsjm\rangle = \frac{eB}{2m_e}\langle nlsjm|(L_z + 2S_z)|nlsjm\rangle$$

The total angular momentum is

$$J = L + S$$

Thus,

$$L_z + 2S_z = J_z + S_z$$

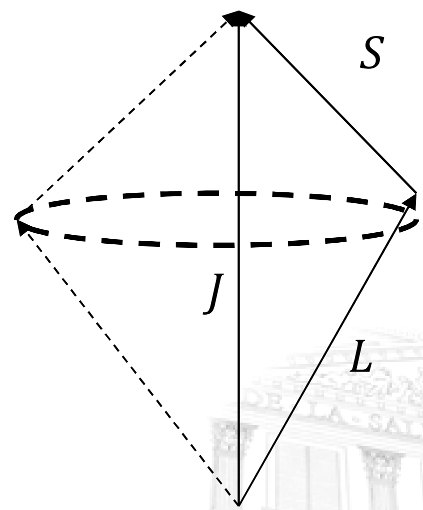
The first term is compatible with the quantum numbers but the second is not

Spin

We note that the total angular momentum is a constant of motion, and that L and S have fixed magnitudes but the vectors precess about the direction of J.

We may then express

$$\vec{S} = (\vec{S} \cdot \hat{J})\hat{J} = \frac{(\vec{S} \cdot \vec{J})}{J^2}\vec{J}$$



Now,

$$L = J - S$$

so

$$L^2 = J^2 + S^2 - 2S \cdot J$$

Thus,

$$S \cdot J = \frac{1}{2}[J^2 + S^2 - L^2]$$

and

$$S_z = \frac{[J^2 + S^2 - L^2]}{2J^2} J_z$$

Weak Field Correction

Therefore, in weak fields

$$E^{(1)}_{(WZ)} = \frac{eB}{2m_e}\left\langle nlsjm\middle|\left(J_z + \frac{[J^2+S^2-L^2]}{2J^2}\,J_z\right)\middle|nlsjm\right\rangle$$

$$= \frac{eB}{2m_e}\left[1 + \frac{j(j+1)+\frac{3}{4}-l(l+1)}{2j(j+1)}\right]m\hbar$$

For $j = l + \frac{1}{2}$,

$$\frac{j(j+1)+\frac{3}{4}-l(l+1)}{2j(j+1)} = \frac{\left(l+\frac{1}{2}\right)\left(l+\frac{3}{2}\right)-l(l+1)+\frac{3}{4}}{2\left(l+\frac{1}{2}\right)\left(l+\frac{3}{2}\right)} = \frac{l^2+2l+\frac{3}{4}-l^2-l+\frac{3}{4}}{(2l+1)\left(l+\frac{3}{2}\right)}$$

$$= \frac{l+\frac{3}{2}}{(2l+1)\left(l+\frac{3}{2}\right)} = \frac{1}{(2l+1)}$$

For $j = l - \frac{1}{2}$,

$$\frac{j(j+1)+\frac{3}{4}-l(l+1)}{2j(j+1)} = \frac{\left(l-\frac{1}{2}\right)\left(l+\frac{1}{2}\right)-l(l+1)+\frac{3}{4}}{2\left(l-\frac{1}{2}\right)\left(l+\frac{1}{2}\right)} = \frac{l^2-\frac{1}{4}-l^2-l+\frac{3}{4}}{2\left(l-\frac{1}{2}\right)\left(l+\frac{1}{2}\right)}$$

$$= \frac{-l+\frac{1}{2}}{\left(l-\frac{1}{2}\right)(2l+1)} = -\frac{1}{(2l+1)}$$

Weak Field Correction

The weak field Zeeman effect is therefore

$$E^{(1)}_{(WZ)} = \frac{eB}{2m_e}\left[1 \pm \frac{1}{(2l+1)}\right] m\hbar$$