

Quantum Mechanics

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Odd and Even Functions



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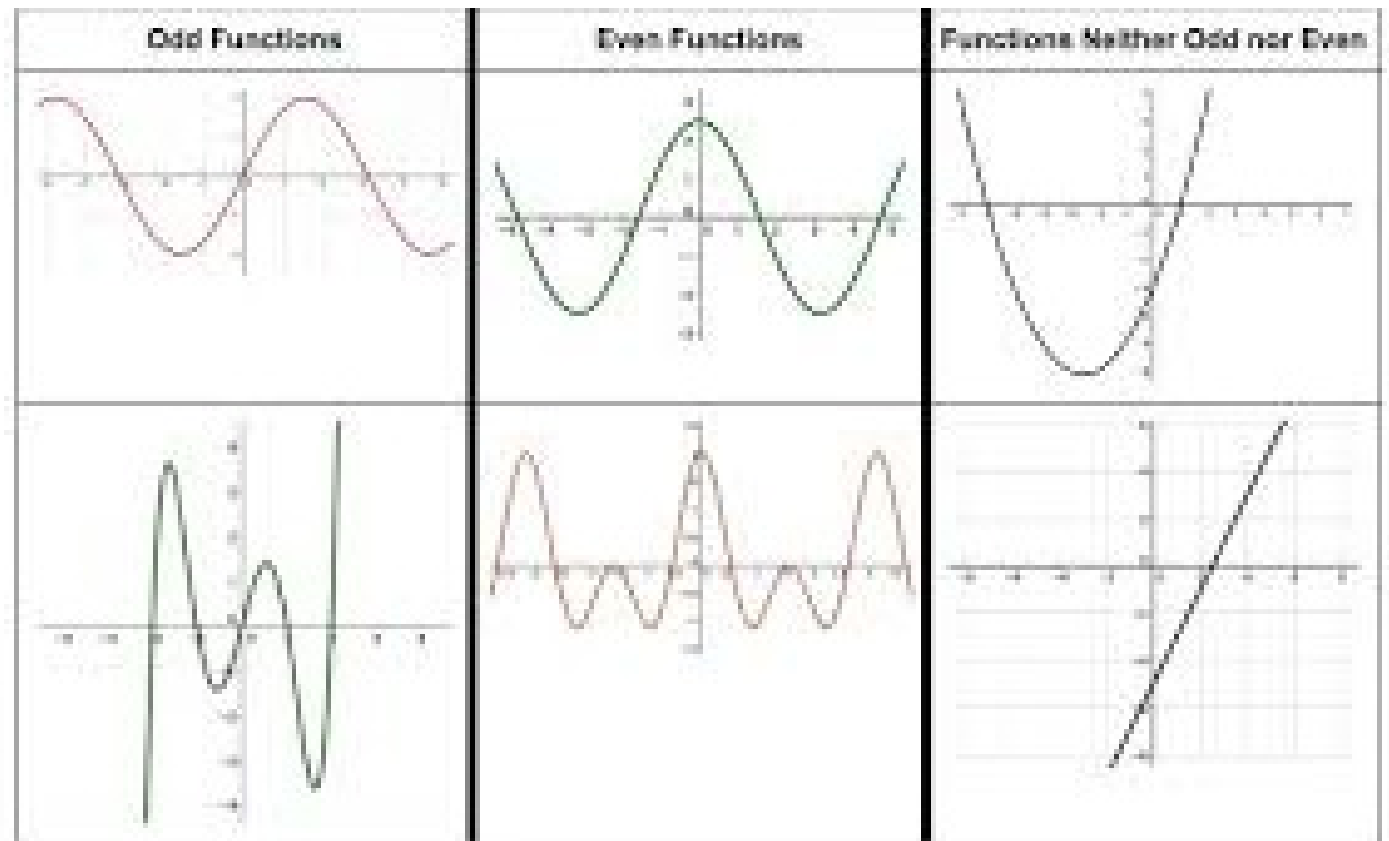
Odd and Even Functions

A function is **odd** if

$$f(-x) = -f(x)$$

It is **even** if

$$f(-x) = f(x)$$



Symmetric Integrals

A symmetric integral can be recast as follows:

$$\int_{-a}^a f(x)dx = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$$

The integral over negative values of x can be rewritten using $x = -|x|$

$$\int_{-a}^0 f(x)dx = -\int_a^0 f(-|x|)d|x| = \int_0^a f(-|x|)d|x|$$



Symmetric Integrals of an Odd Function

For an odd function,

$$\int_{-a}^0 f(x)dx = \int_0^a f(-|x|)d|x| = -\int_0^a f(|x|)d|x|$$

Thus,

$$\int_{-a}^a f(x)dx = -\int_0^a f(|x|)d|x| + \int_0^a f(x)dx$$

Now, $|x| \geq 0$. So the first integral on the right is over the same range of x as the second integral. Hence, for an odd function,

$$\int_{-a}^a f(x)dx = -\int_0^a f(x)dx + \int_0^a f(x)dx = 0$$



Symmetric Integrals of an Even Function

For an even function,

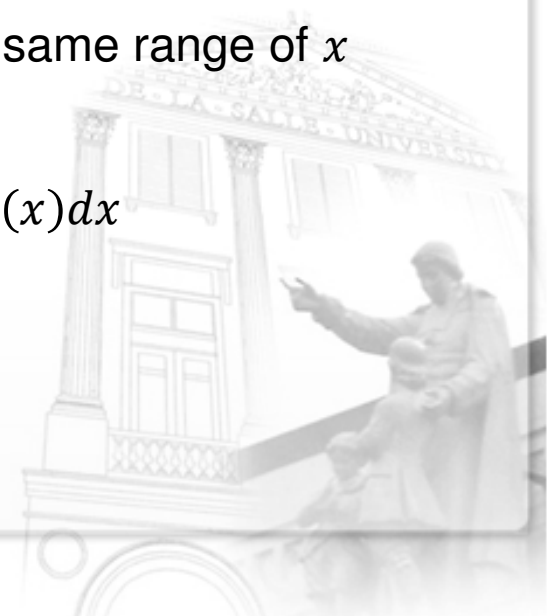
$$\int_{-a}^0 f(x)dx = \int_0^a f(-|x|)d|x| = \int_0^a f(|x|)d|x|$$

Thus,

$$\int_{-a}^a f(x)dx = \int_0^a f(|x|)d|x| + \int_0^a f(x)dx$$

Now, $|x| \geq 0$. So the first integral on the right is over the same range of x as the second integral. Hence, for an even function,

$$\int_{-a}^a f(x)dx = \int_0^a f(x)dx + \int_0^a f(x)dx = 2 \int_0^a f(x)dx$$



Examples

$$\int_{-a}^a x dx = \left[\frac{x^2}{2} \right]_{-a}^{-a} = \frac{a^2}{2} - \frac{(-a)^2}{2} = 0$$

$$\int_{-a}^a 3x^2 dx = [x^3]_{-a}^{-a} = a^3 - (-a)^3 = 2a^3$$

$$\int_{-a}^a 4x^3 dx = [x^4]_{-a}^{-a} = a^4 - (-a)^4 = 0$$

$$\int_{-a}^a 5x^4 dx = 2 \int_0^a 5x^4 dx = 2a^5$$

$$\int_{-a}^a \cos x dx = 2 \int_0^a \cos x dx = 2 \sin a$$

$$\int_{-a}^a \sin x dx = 0$$

$$\int_{-a}^a x \cos x dx = 0$$

