

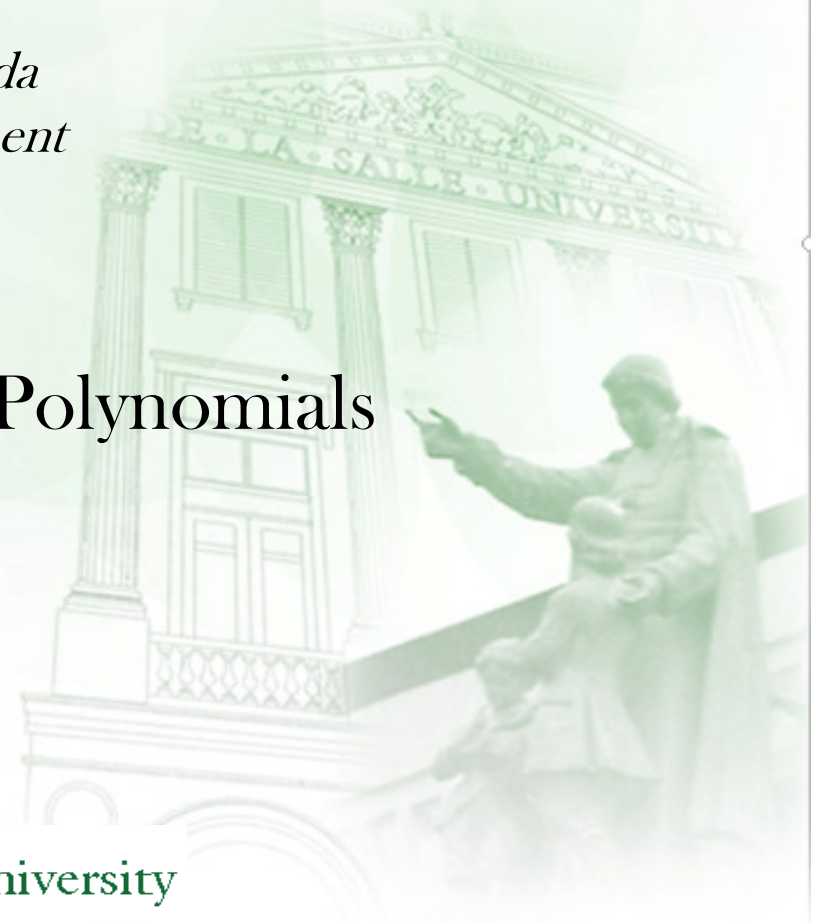
Quantum Mechanics 2

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Associated Laguerre Polynomials Properties



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Associated Laguerre Equation

The Associated Laguerre Equation

$$\rho \frac{d^2 L}{d\rho^2} + (k + 1 - \rho) \frac{dL}{d\rho} + sL = 0$$

has the solutions

$$L_s^k(\rho) = \sum_{m=0}^s (-1)^m \frac{(s+k)!}{(s-m)! m! (k+m)!} \rho^m$$

The Associated Laguerre polynomials L_s^k may also be evaluated from the Laguerre Polynomials through

$$L_s^k(\rho) = (-1)^k \frac{d^k}{d\rho^k} L_{s+k}(\rho)$$

The Laguerre polynomials L_n satisfies the equation

$$\rho \frac{d^2 L}{d\rho^2} + (1 - \rho) \frac{dL}{d\rho} + nL = 0$$



Recursion and Orthogonality

Alternatively, the Associated Laguerre polynomials can be evaluated using Rodrigues' formula

$$L_s^k(\rho) = \frac{1}{s!} e^\rho \rho^k \frac{d^s}{d\rho^s} (e^{-\rho} \rho^{s+k})$$

A potent relation is the recursion relation

$$(s + 1)L_{s+1}^k(\rho) = (2s + k + 1 - \rho)L_s^k(\rho) - (s + k)L_{s-1}^k(\rho)$$

and its derivative

$$\rho \frac{dL_s^k(\rho)}{d\rho} = sL_s^k(\rho) - (s + k)L_{s-1}^k(\rho)$$

Of particular importance is the orthogonality condition

$$\int_0^\infty e^{-\rho} \rho^k L_s^k(\rho) L_{s'}^k(\rho) d\rho = \frac{(s + k)!}{s!} \delta_{ss'}$$



Other Useful Relations

Another useful relations is

$$L_s^k(0) = \frac{(s+k)!}{s!k!}$$

The recursion relation can be rearranged as

$$\rho L_s^k(\rho) = (2s+k+1)L_s^k(\rho) - (s+k)L_{s-1}^k(\rho) - (s+1)L_{s+1}^k(\rho)$$

Thus,

$$\int_0^\infty e^{-\rho} \rho^{k+1} L_s^k(\rho) L_{s'}^k(\rho) d\rho = \int_0^\infty e^{-\rho} \rho^k [(2s+k+1)L_s^k(\rho) - (s+k)L_{s-1}^k(\rho) - (s+1)L_{s+1}^k(\rho)] L_{s'}^k(\rho) d\rho$$

Applying the orthogonality condition, we get have

$$\int_0^\infty e^{-\rho} \rho^{k+1} L_s^k(\rho) L_s^k(\rho) d\rho = \frac{(s+k)!}{s!} (2s+k+1)$$

