# Quantum Mechanics

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## Gaussian Integral

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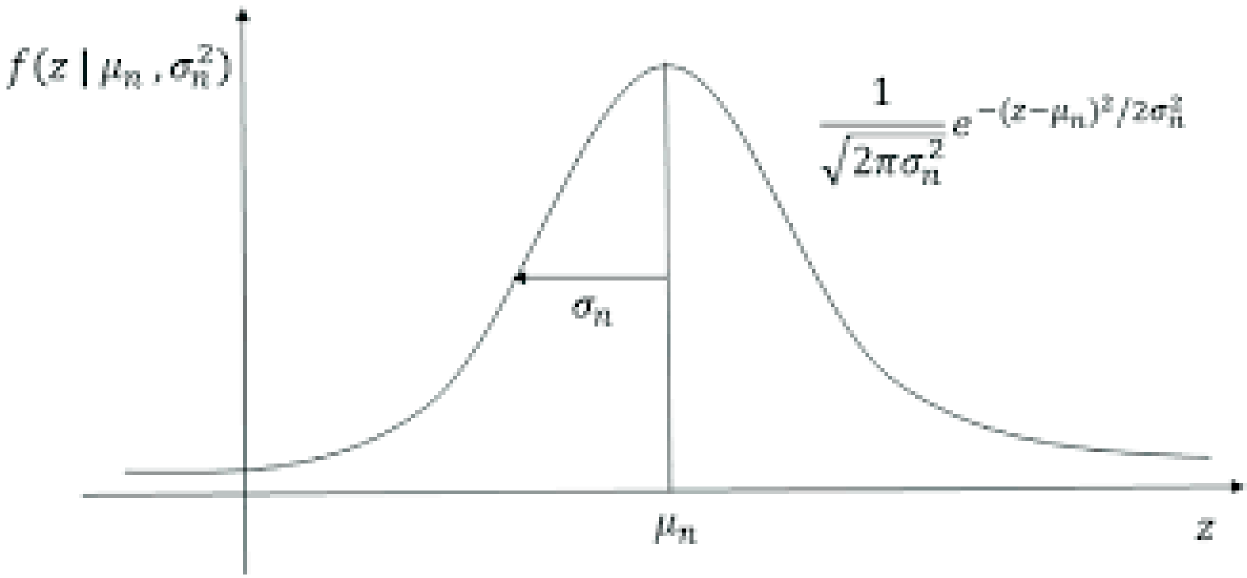
# The Gaussian Function

The exponential of a square

$$e^{-\alpha x^2}$$

is called the Gaussian function, a normal distribution, or the Bell Curve.

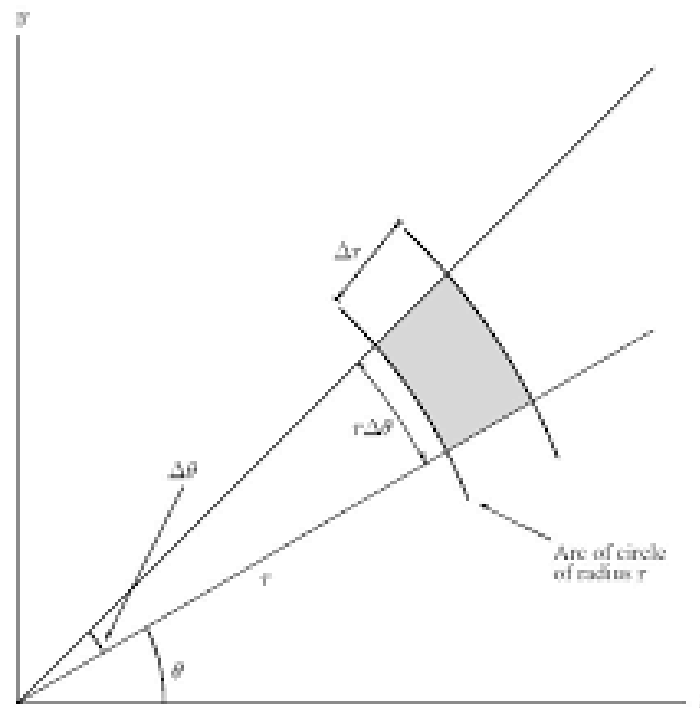


# Gaussian Integral

The Gaussian Integral

$$I = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx$$

may be evaluated as follows:




$$I^2 = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx \int_{-\infty}^{\infty} e^{-\alpha y^2} dy = \int e^{-\alpha r^2} da$$

$$= \int_0^{\infty} r dr e^{-\alpha r^2} \int_0^{2\pi} d\theta$$

Let $u = r^2$,

$$I^2 = 2\pi \int_0^{\infty} \frac{1}{2} du e^{-\alpha u} = -\frac{\pi}{\alpha} [e^{-\alpha u}]_0^{\infty}$$

$$= -\frac{\pi}{\alpha}(0 - 1) = \frac{\pi}{\alpha}$$

Thus,

$$I = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$