Quantum Mechanics

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Inverse

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Inverse

The inverse of a matrix A is the matrix A^{-1} which when multiplied to A yields the unit matrix

$$AA^{-1} = A^{-1}A = 1$$

For a square matrix, the elements of the inverse are

$$(A^{-1})_m^n = \frac{C_n^m}{\Delta}$$

where Δ is the determinant of A, and C_n^m is the cofactor of the element A_n^m of the matrix A (note that this is the transpose).

Example

For

$$A = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 2 & 6 \\ -5 & 3 & 4 \end{pmatrix}$$

The determinant is

$$\Delta = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & 6 \\ -5 & 3 & 4 \end{vmatrix} = 30 + 9 + 30 + 4 = 73$$

The cofactors of each element of A are

$$C_1^1 = \begin{vmatrix} 2 & 6 \\ 3 & 4 \end{vmatrix} = 8 - 18 = -10$$

$$C_1^2 = -\begin{vmatrix} 1 & 6 \\ -5 & 4 \end{vmatrix} = -(4 + 30) = -34$$

$$C_1^3 = \begin{vmatrix} 1 & 2 \\ -5 & 3 \end{vmatrix} = 3 + 10 = 13$$

Example

$$C_2^1 = -\begin{vmatrix} -1 & 3 \\ 3 & 4 \end{vmatrix} = -(-4-9) = 13$$

$$C_2^2 = \begin{vmatrix} 0 & 3 \\ -5 & 4 \end{vmatrix} = 15$$

$$C_2^3 = -\begin{vmatrix} 0 & -1 \\ -5 & 3 \end{vmatrix} = 5$$

$$C_3^1 = \begin{vmatrix} -1 & 3 \\ 2 & 6 \end{vmatrix} = -6-6 = -12$$

$$C_3^2 = -\begin{vmatrix} 0 & 3 \\ 1 & 6 \end{vmatrix} = 3$$

$$C_3^3 = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1$$

Thus,

$$A^{-1} = \frac{1}{73}\begin{pmatrix} -10 & 13 & -12 \\ -34 & 15 & 3 \\ 13 & 5 & 1 \end{pmatrix}$$

Verification

To verify,

$$A^{-1}A = \frac{1}{73}\begin{pmatrix} -10 & 13 & -12 \\ -34 & 15 & 3 \\ 13 & 5 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 3 \\ 1 & 2 & 6 \\ -5 & 3 & 4 \end{pmatrix}$$

$$= \frac{1}{73}\begin{pmatrix} 13+60 & 10+26-36 & -30+78-48 \\ 15-15 & 34+30+9 & -102+90+12 \\ 5-5 & -13+10+3 & 39+30+4 \end{pmatrix} = \frac{1}{73}\begin{pmatrix} 73 & 0 & 0 \\ 0 & 73 & 0 \\ 0 & 0 & 73 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$