Quantum Mechanics

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Hyperbolic Functions

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Euler Relations

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

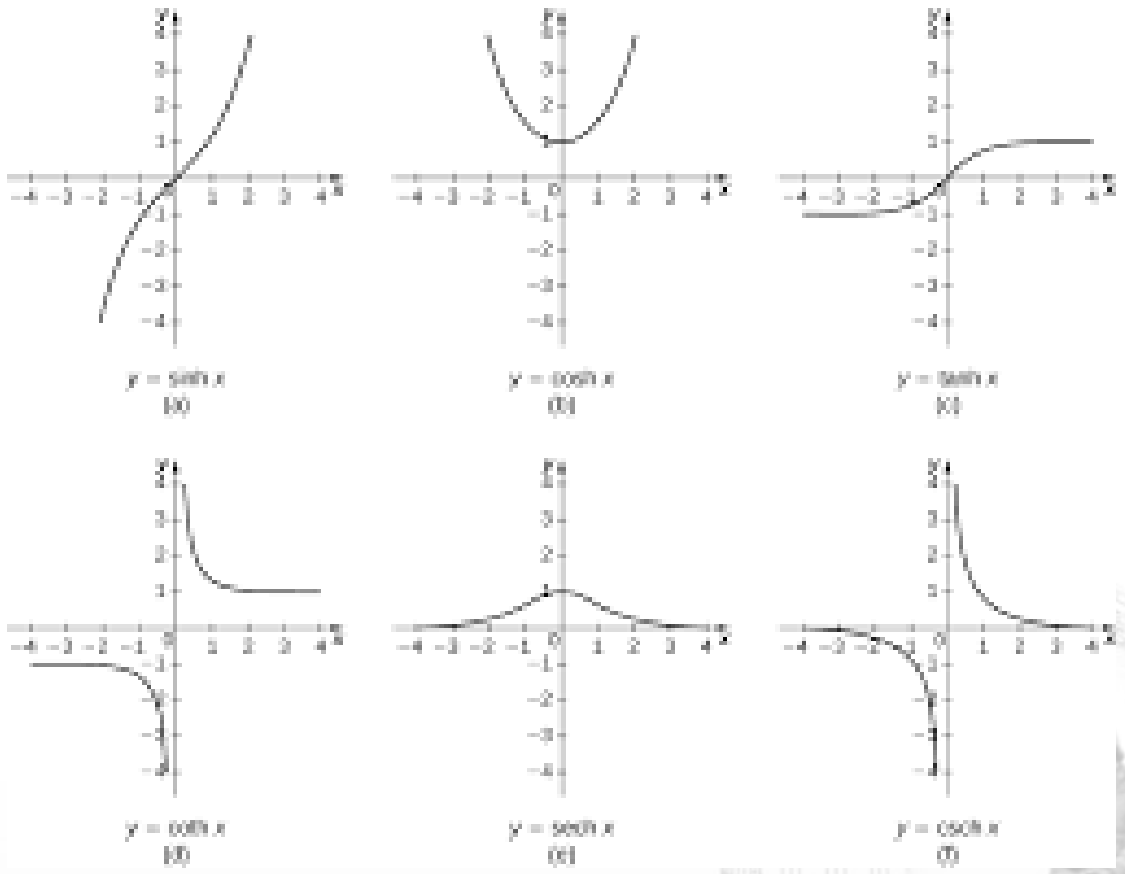
Thus,

$$\sinh ix = i \sin x$$

$$\cosh ix = \cos x$$

$$\sin ix = i \sinh x$$

$$\cos ix = \cosh x$$



"Pythagorean" Theorem

$$(\sinh x)^2 = \frac{e^{2x} - 2 + e^{-2x}}{4}$$

$$(\cosh x)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$$

Thus,

$$(\cosh x)^2 - (\sinh x)^2 = 1$$

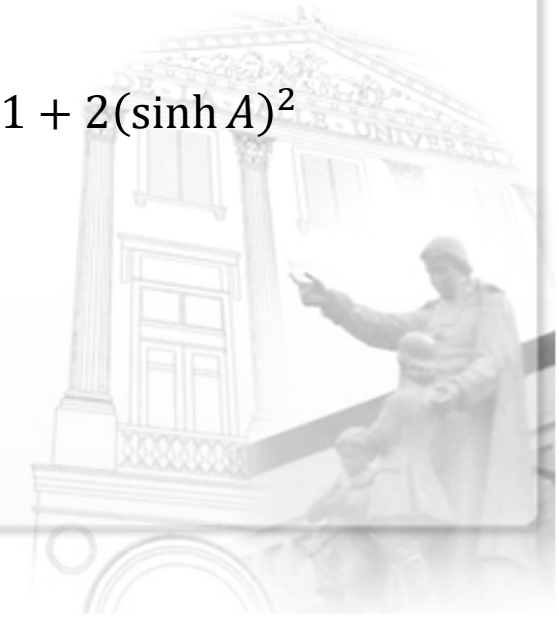
Addition Formulas

$$\sin h(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$$

$$\cosh(A \pm B) = \cosh A \cosh B \ \pm \sinh A \sinh B$$

$$\sinh 2A = 2 \sinh A \cosh A$$

$$\cosh 2A = (\cosh A)^2 + (\sinh A)^2 = 2\,(\cosh A)^2 - 1 = 1 + 2(\sinh A)^2$$



Sum and Difference

$$\sinh A + \sinh B = 2\sinh\tfrac{1}{2}(A+B)\cosh\tfrac{1}{2}(A-B)$$

$$\sinh A - \sinh B = 2\cosh\tfrac{1}{2}(A+B)\sinh\tfrac{1}{2}(A-B)$$

$$\cosh A + \cosh B = 2\cosh\tfrac{1}{2}(A+B)\cosh\tfrac{1}{2}(A-B)$$

$$\cosh A - \cosh B = 2\sinh\tfrac{1}{2}(A+B)\sinh\tfrac{1}{2}(A-B)$$

Products

$$\sinh A \sinh B = \frac{1}{2}[\cosh(A+B) - \cosh(A-B)]$$

$$\cosh A \cosh B = \frac{1}{2}[\cosh(A+B) + \cosh(A-B)]$$

$$\sinh A \cosh B = \frac{1}{2}[\sinh(A+B) + \sin h(A-B)]$$

Powers of Sinh

$$(\sinh A)^2 = \frac{1}{2}\cosh 2A - \frac{1}{2}$$

$$(\sinh A)^3 = -\frac{3}{4}\sinh A + \frac{1}{4}\sinh 3A$$

$$(\sinh A)^4 = \frac{3}{8} - \frac{1}{2}\cosh 2A + \frac{1}{8}\cosh 4A$$

Powers of Cosh

$$(\cosh A)^2 = \frac{1}{2} + \frac{1}{2}\cosh 2A$$

$$(\cosh A)^3 = \frac{3}{4}\cosh A + \frac{1}{4}\cosh 3A$$

$$(\cosh A)^4 = \frac{3}{8} + \frac{1}{2}\cosh 2A + \frac{1}{8}\cosh 4A$$